THEMATIC SESSION: LOGIC

Cardinal sequences in topology

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If X is a topological space and α is an ordinal, we define the α^{th}- Cantor-Bendixson derivative of X by transfinite induction on α as follows:
(a) $X^0 = X$,
(b) if $\alpha = \beta + 1$, $X^\alpha =$ set of accumulation points of X^β,
(c) if α is a limit ordinal, $X^\alpha = \bigcap\{X^\beta : \beta < \alpha\}$.
The Cantor-Bendixson derivatives of a topological space X form a decreasing sequence, i.e. we have $X^\alpha \supset X^\beta$ if $\alpha < \beta$.
Then, for every ordinal α we define he α^{th}-level of X as $I_\alpha(X) = X^\alpha \setminus X^{\alpha+1}$. So, $I_\alpha(X) =$ the set of isolated points of the subspace $X^\alpha = X \setminus \bigcup\{I_\beta(X) : \beta < \alpha\}$.
We say that a topological space X is scattered, if there is an ordinal α such that $X^\alpha = \emptyset$. If X is a scattered space, we define the cardinal sequence of X as the sequence formed by the cardinalities of the infinite Cantor-Bendixson levels of X, i.e.

$$\mathrm{CS}(X) = \langle |I_\alpha(X)| : \alpha < \delta \rangle$$

where δ is the least ordinal γ such that $I_\gamma(X)$ is finite.
Many authors have studied the possible sequences of infinite cardinals that can arise as the cardinal sequence of some scattered space. In this talk we will show some of the most relevant results on this topic