

## THEMATIC SESSION: LOGIC

**Structural and universal completeness in algebra and logic**

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This talk is about the interplay of the notions of derivability and admissibility for the rules of a logic, the latter seen as a substitution-invariant consequence relation among formulas written over some algebraic language. In particular, a rule is admissible in a logic  $L$  if every substitution making the premises a theorem of  $L$ , also makes the conclusions a theorem of  $L$ . For a logic  $L$ , being structurally complete then means that all the admissible rules of  $L$  are derivable in  $L$ , or equivalently, each of its proper extensions admits new theorems. If the logic  $L$  is algebraizable with equivalent algebraic semantics a quasivariety  $Q$ , the structural completeness of  $L$  can be studied algebraically;  $Q$  (and so  $L$ ) is structurally complete if every admissible quasiequation is valid in  $Q$  (where a quasiequation is admissible if every substitution making the premises valid in  $Q$ , also makes the conclusions valid in  $Q$ ). Universal completeness moves this same notion to multiple-conclusion rules, or clauses instead of quasiequations. Now, for instance, Classical Logic and Boolean algebras are structurally complete but not universally complete, while Intuitionistic Logic and Heyting algebras are not even structurally complete. The latter are, however, passively structurally complete: all the quasiequations that are passive (i.e., such that there is no substitution making their premises valid) are valid. We will present some bridge theorems between the logical and algebraic notions of structural and universal completeness and their relevant weakenings; in particular, we give new algebraic characterizations of quasivarieties that are passively structurally complete, and actively universally complete. Finally, we show some consequences for varieties of bounded lattices, and for the equivalent algebraic semantics of substructural logics; e.g. we will see that within the large container of substructural logics with weakening, a logic is passively structurally complete if and only if every classical contradiction is explosive in it.

This is joint work with Paolo Aglianò, from the University of Siena (Italy).