THEMATIC SESSION: Dynamical Systems

Attractive invariant circles through elliptic methods

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Studying general perturbations of a dissipative twist map depending on two parameters, a frequency ν and a dissipation η, the existence of a Cantor set $\mathcal{C}$ of curves in the (ν, η) plane such that the corresponding equation possesses a Diophantine quasi-periodic invariant circle can be deduced, up to small values of the dissipation, as direct consequence of a normal form Theorem in the spirit of Rüssmann and the "elimination of parameters" technique. These circles are normally hyperbolic as soon as $\eta \neq 0$, which implies that the equation still possesses a circle of this kind for values of the parameters belonging to a neighborhood $\mathcal{V}$ of this set of curves. Obviously, the dynamics on such invariant circles is no more controlled and may be generic, but the normal dynamics is controlled in the sense of their basins of attraction.
As it is expected, by classical graph-transform method we are able to determine a first rough region where the normal hyperbolicity prevails and a circle persists, for a strong enough dissipation $\eta \sim O(\sqrt{\epsilon})$, ϵ being the size of the perturbation. Then, through normal-form techniques, we shall enlarge such regions and determine such a (conic) neighborhood $\mathcal{V}$, up to values of dissipation of the same order as the perturbation, by using the fact that the proximity of the set $\mathcal{C}$ allows, thanks to Rüssmann's translated curve Theorem, to introduce local coordinates of the type (dissipation, translation) similar to the ones introduced by Chenciner in [1].

References

[1] A. Chenciner. Bifurcations de points fixes elliptiques. I. Courbes invariantes. *Inst. Hautes Études Sci. Publ. Math.*, 61:67–127, 1985.