THEMATIC SESSION: Analysis and PDEs

Hardy–Littlewood maximal operators on trees with bounded geometry

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The centred Hardy–Littlewood maximal operator $\mathcal{M}$ on homogeneous trees is of weak type $(1, 1)$ and hence bounded on L^p for every $p > 1$ [CMS, NT]. In this talk we enlarge the view to $\Upsilon_{a,b}$, the family of trees on which the valence of each vertex can oscillate between the values a and b. We will discuss the following unexpected dichotomy concerning trees $\mathfrak{T} \in \Upsilon_{a,b}$: if $b > a^2$ the range of $p \in (1, \infty)$ for which $\mathcal{M}$ is bounded on $L^p(\mathfrak{T})$ cannot depend solely on a and b, but also depends on the locations of the vertices with different valence within $\mathfrak{T}$, while for $a < b \leq a^2$, $\mathcal{M}$ is bounded on $L^p(\mathfrak{T})$ for every $p > \log_a b$, independently of the locations of the vertices. In the latter case, the range of p can be proved to be sharp, meaning that for $p \leq \log_a b$ one can always locate the vertices of $\mathfrak{T}$ in such a way that $\mathcal{M}$ is unbounded $L^p(\mathfrak{T})$. On the other hand, if the vertices of $\mathfrak{T} \in \Upsilon_{a,b}$ with different valence are well distributed one can recover the weak $(1, 1)$ boundedness of $\mathcal{M}$ on $\mathfrak{T}$, as in the homogeneous case. Time permitting, a few words will be devoted to the same problem for the uncentred maximal operator.

The talk is based on a joint work with S. Meda, F. Santagati e M. Vallarino.

References

[CMS] M. Cowling, S. Meda, A. G. Setti, A weak type $(1, 1)$ estimate for a maximal operator on a group of isometries of homogeneous trees, *Coll. Math.* **118** (2010), 223–232.

[NT] A. Naor and T. Tao, Random martingales and localization of maximal inequalities, *J. Funct. Anal.* **259** (2010), 731–779.