THEMATIC SESSION: Dynamical Systems

On a question of Newtonian mechanics asked by Mark Levi

Andrea Venturelli (Université d'Avignon)

In 2003, Mark Levi asked the following question: given a mechanical newtonian system on the plane $\mathbb{R}^2$, governed by a potential U, what can be said on U if we know that each level curve of U can be parametrized so that it is a solution of the associated newtonian system? A classical exemple of a potential satisfying this assumption is a potential with a radial symmetry. It is natural to ask if there are others exemples. We will see that the answer depends on some additional assumptions about U. Specifically, we will see that if U is real analytic then U is necessarily radial. If U is assumed to be smooth and $\mathrm{Crit}(U)$ is supposed to be totally path disconnected, the conclusion is still the same. But without this assumption on the set of critical points, there are exemples of non radial smooths potentials U such that each level curve of U can be parametrized so that it is a solution of the associated newtonian system. This problem is related to en evolution parabolic equation on the set of convex curves on $\mathbb{R}^2$ called inverse curvature flow.

It is a joint work with Philippe Bolle and Marco Mazzucchelli.