THEMATIC SESSION:

A shape optimization problem in relativistic quantum mechanics

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Dirac operators defined on domains of the Euclidean space are used in relativistic quantum mechanics to describe particles that are confined in a region. A remarkable example is the MIT bag operator, used to model confinement of quarks in hadrons, and a fundamental topic in mathematical physics concerns the analysis of the spectral gap and its associated shape optimization problem. This consists on minimizing the first squared eigenvalue among all domains with prescribed volume, and it is conjectured that the ball is the optimal domain.

In this talk I will describe a recent work —in collaboration with N. Arrizabalaga, A. Mas, and L. Vega — in which we propose an approach towards this open problem. We have studied a family of Dirac operators defined on a domain of the three-dimensional euclidean space and with boundary conditions that depend on a real parameter. This family contains the MIT bag operator (when the parameter is zero), while some well-known operators arise in the limits as the parameter goes to plus or minus infinity. We parametrize the spectrum of the family of operators through a collection of increasing smooth curves, and we study the limit operators.

Thanks to this analysis, we manage to establish (for large values of the parameter) the optimality of the ball for the associated shape optimization problem. This is expected to hold for all the parametrization and thus solve the open problem for the MIT bag operator.