

THEMATIC SESSION: Logic

Profinite Heyting algebras and the representation problem for Esakia spaces

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A poset is said to be "representable" when it is isomorphic to the prime spectrum of a bounded distributive lattice or, equivalently, of a commutative ring with unit. The problem of characterizing the representable posets was raised by Grätzer and Kaplansky in lattice theory and general algebra, respectively.

While it is an immediate consequence of Priestley duality that a poset is representable precisely when it can be endowed with a topology that turns it into a Priestley space, the problem of obtaining a more concrete description of the representable posets seems to elude easy solutions.

In this talk, we will consider the related problem of describing the posets isomorphic to the prime spectra of Heyting algebras. We call these posets "Esakia representable" because they coincide with the posets that can be endowed with a topology that turns them into Esakia spaces. On the one hand, we characterize the Esakia representable well-ordered trees. On the other hand, we employ the theory of profinite Heyting algebras to obtain a description of the top part of Esakia representable posets. When phrased in purely algebraic terms, our main result takes the form of a description of the varieties of Heyting algebras whose profinite members are profinite completions.

This talk is based on joint work with G. Bezhanishvili, N. Bezhanishvili, and M. Stronkowski.