THEMATIC SESSION: Discrete Mathematics

Sensitivity in Cayley graphs

Ignacio García Marco (Universidad de La Laguna)

In 2019, Huang proved that the sensitivity and the degree of a boolean function are polynomially related, solving an outstanding foundational problem in theoretical computer science: the Sensitivity Conjecture of Nisan and Szegedy. The key point of his argument is the proof that every set of more than half the vertices of the hypercube graph induces a subgraph of high maximum degree. Huang asked whether similar results can be obtained for other highly symmetric graphs.

In this work we first prove that this result cannot be extended to general Cayley graphs. We present infinite families of Cayley graphs of groups of unbounded degree that contain induced subgraphs of maximum degree 1 on more than half the vertices.

Second, we propose Coxeter groups as a suitable generalization of the hypercube with respect to Huang's question. We support our proposal with some partial results plus a large amount of computer assisted experiments.

Finally, we provide examples of cube-free Cayley graphs where every induced subgraph on more than half the vertices has high maximum degree. Interestingly, these examples rely on point-line incidence results of projective planes over a finite field.

This is joint work with Kolja Knauer.