THEMATIC SESSION: New trends in algebraic geometry

Embedded deformations of curves with maximal variation of Hodge structure

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Given a family of complex (smooth projective) manifolds, the dimension of the corresponding subset of the local moduli space gives a first measure of how far it is from being trivial. This can be then refined taking into account how much the Hodge structures of the fibres change, which leads to the notion of maximal (infinitesimal) variation of Hodge structure (IVHS).

In the case of families of curves, results of Lee-Pirola and of myself with Torelli imply in particular that a general deformation of any curve has maximal IVHS. However this is not so clear if one wants the deformation to keep some further structure, such as the gonality of the curve or an embedding into a given surface. For example, it was only recently proved by Favale and Pirola that every smooth plane curve admits a deformation as a plane curve with maximal IVHS, and the question remains open for deformations of curves inside any other surface.

In this talk I will present a joint work in progress with Sara Torelli extending this result to curves in $\mathbb{P}^1 \times \mathbb{P}^1$, which turns out to be surprisingly more involved than the plane case.