

THEMATIC SESSION: Logic

Free algebras and coproducts in varieties of Gödel algebras

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The Gödel-Dummett logic is a point of contact of intuitionistic and fuzzy logic. Indeed, it is obtained by adding the prelinearity axiom $(p \rightarrow q) \vee (q \rightarrow p)$ to the intuitionistic calculus, but it is also the fuzzy logic with truth values in the real unit interval induced by the minimum t-norm. The algebraic semantics for the propositional Gödel-Dummett logic is provided by Gödel algebras, which are the Heyting algebras validating the prelinearity axiom.

In this talk, we see how to dually describe free Gödel algebras. More precisely, we employ Priestley and Esakia dualities to describe the Esakia dual of the Gödel algebra free over a distributive lattice L in terms of the Priestley dual of L . As a consequence, we obtain a dual description of free Gödel algebras, which allow us to show that any free Gödel algebra is a bi-Heyting algebra. Using similar techniques, we also describe the Esakia duals of coproducts in the variety of Gödel algebras. These results generalize some well-known descriptions of finitely generated free Gödel algebras and coproducts of finite Gödel algebras due to Aguzzoli, D'Antona, Gerla, Grigolia, and Marra. In the last part of the talk, we adapt these dual descriptions to all subvarieties of the varieties of Gödel algebras and of symmetric Gödel algebras.